Structure of Dimensions/Revolution of Dimensions (Classical and Fractal) in Education and Science

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ABSTRACT. As teachers we are challenged to present our ideas and the accompanying data using an appropriate theoretical background. To achieve this, the integration of several disciplines, independent of each other, may be necessary. One of the most important problems to achieve this aim is to give a suitable and known mathematical background to a given problem, integrating many disciplines, and then apply information technology.

Therefore, it seems to be essential to rethink, complete and redefine our ideas about the concept of dimensions, placing them in a more practical context.

In this presentation, the classical ideas and the historical development of the concept of dimension will be discussed, using mainly mathematical and physical examples. Present day descriptions will be discussed, expanding our discussion to the future. The mathematical definition and practical use of fractional dimension will also be discussed.

Key-Words: Dimensions, Fractals, Image Processing, Fractal Dimension, Spectral Fractal Dimension

1. Introduction

In the IT-aimed research-developments of present days there are more and more processes that derive from fractals, programs containing fractal based algorithms as well as their practical results. Our topic is the introduction of ways of application of fractal dimension, together with the mathematical extension of fractal dimension, the description of a new algorithm based on the mathematical concept, and the introduction of its practical applications.

2. The Fractal Dimension

Fractal dimension is a mathematical concept which belongs to fractional dimensions. Among the first mathematical descriptions of self similar formations can be found von Koch's descriptions of snowflake curves (around 1904) [14]. With the help of fractal dimension it can be defined how irregular a fractal curve is. In general, lines are one dimensioned, surfaces are two dimensioned and bodies are three dimensioned. Let us take a very irregular curve however which wanders to and from on a surface (e.g. a sheet of paper) or in the three dimension space. In practice [1], [2], [3], [4] [8], [9], [10], [11], [12], [13], [14], [15] we know several curves like this: the roots of plants, the branches of trees, the branching network of blood vessels in the human body, the lymphatic system, a network of roads etc. Thus, irregularity can also be considered as the extension of the concept of dimension. The dimension of an irregular curve is between 1 and 2, that of an irregular surface is between 2 and 3. The dimension of a fractal curve is a number that characterises how the distance grows between two given points of the curve while increasing resolution. That is, while the topological dimension of lines and surfaces is always 1 or 2, fractal dimension can also be in between. Real life curves and surfaces are not real fractals, they derive from processes that can form configurations only in a given measure.

Thus dimension can change together with resolution. This change can help us characterize the processes that created them.

The definition of a fractal, according to Mandelbrot is as follows: A fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension [12].

The theoretical determination of the fractal dimension [1]: Let (X,d) be a complete metric space. Let $A \in H(X)$. Let $N(e)$ denote the minimum number of balls of radius e needed to cover A. If

$$FD = \lim_{e \to 0} \left\{ Sup \left\{ \frac{LnN(\bar{e})}{Ln(1/\bar{e})} : \bar{e} \in (0,e) \right\} \right\} \qquad (1)$$

exists, then FD is called the fractal dimension of A.
The general measurable definition of fractal dimension (FD) is as follows:

$$FD = \frac{\log \frac{L_2}{L_1}}{\log \frac{S_1}{S_2}} \qquad (2)$$

where L_1 and L_2 are the measured length on the curve, S_1 and S_2 are the size of the used scales (that is, resolution).
There have been several methods developed that are suitable for computing fractal dimension as well. [14] - Table 1.

2.1 MEASURING FRACTAL DIMENSION

Fractal dimension, which can be the characteristic measurement of mainly the structure of an object in a digital image [15], [12], [4], [1], can be computed applying the Box counting as follows:

1. Segmentation of image
2. Halving the image along vertical and horizontal symmetry axis
3. Examination of valuable pixels in the box
4. Saving the number of boxes with valuable pixels
5. Repeat 2-4 until shorter side is only 1 pixel
   To compute dimension, the general definition can be applied to the measured data like a function (number of valuable pixels in proportion to the total number of boxes).

Methods	**Main facts**
Least Squares Approximation	theoretical
Walking-Divider	practical to length
Box Counting	most popular
Prism Counting	for a one dimensional signals
Epsilon-Blanket	to curve
Perimeter-Area relationship	to classify different types images
Fractional Brownian Motion	similar box counting

Power Spectrum	digital fractal signals
Hybrid Methods	calculate the fractal dimension of 2D using 1D methods

Table 1: Methods of Computing Fractal Dimensions

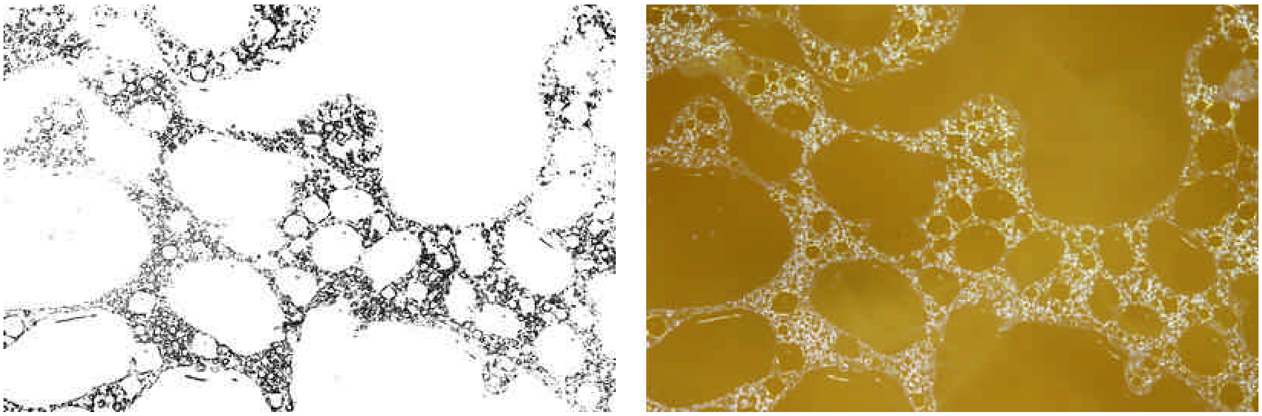

Figure 1: The fractal dimension measured with the help of the Box counting of the two images is the same (FD=1.99), although the two images are different in shades of colour.

3. Spectral Fractal Dimension

Nearly all of the methods in Table 1 measure structure. Neither the methods in Table 1 nor the definition and process described above gives (enough) information on the (fractal) characteristics of colours, or shades of colours. Fig. 1 gives an example.

Measuring with the help of the Box counting the image on the right and the one on the left have the same fractal dimension (FD=1.99) although the one on the left is a black and white (8 bit) image whereas the other one on the right is a 24-bit coloured image containing shades as well - the original images can be found at www.georgikon.hu/digkep/sfd/index.htm, [16] -, so there is obviously a significant difference in the information they contain. How could the difference between the two images be proven using measurement on the digital images? Let spectral fractal dimension (SFD) be:

$$SFD = \frac{\log \frac{L_{S2}}{L_{S1}}}{\log \frac{S_{S1}}{S_{S2}}} \qquad (3)$$

where L_{S1} and L_{S2} are measured spectral length on N-dimension colour space, S_{S1} and S_{S2} are spectral metrics (spectral resolution of the image).

In practice, N={1, 3, 4, 6, 32, 79}, where

- N=1 black and white or greyscale image,
- N=3 RGB, YCC, HSB, IHS colour space image,

- N=4 traditional colour printer CMYK space image
- N=6 photo printer CC_pMM_pYK space image or Landsat ETM satellite image
- N=32 DAIS7915 VIS_NIR or DAIS7915 SWIP-2 sensors
- N=79 DAIS7915 all

In practice the measure of spectral resolution can be equalled with the information theory concept of {S_i=1, …, S_i=16, where i=1 or i=2} bits.

Typical spectral resolution:

- Threshold image -1 bit
- Greyscale image - 2-16 bits
- Colour image - 8-16 bits/bands

On this basis, spectral computing is as follows:

1. Identify which colour space the digital image is
2. Establish spectral histogram in the above space
3. Half the image as spectral axis
4. Examine valuable pixels in the given N-dimension space part (N-dimension spectral box)
5. Save the number of the spectral boxes that contain valuable pixels
6. Repeat steps 3-5 until one (the shortest) spectral side is only one (bit).

In order to compute dimension, the definition of spectral fractal dimension can be applied to the measured data like a function (number of valuable spectral boxes in proportion to the whole number of boxes), computing with simple mathematical average as follows:

$$SFD_{measured} = \frac{3 \times \sum_{j=1}^{N-1} \frac{\log(BM_j)}{\log(BT_j)}}{N-1} \qquad (4)$$

where

- BM_j - number of spectral boxes containing valuable pixels in case of j-bits
- BT_j – total number of possible spectral boxes in case of j-bits

During computing:

1. Establish the logarithm of the ratio of BM/BT to each spectral halving
2. Multiply the gained values with N (index of dimension)
3. Find the mathematical average of the previously gained values

On this basis, the measured SFD of the two images introduced in figure 1 show an unambiguous difference ($SFD_{\text{left side image}}$=1.21, $SFD_{\text{right side image}}$=2.49).

The SFD results measured by the program are invariant for identical scale pixels with different geometric positions in case the number of certain scales is the same and shade of colour is constant.

Successful practical application of SFD at present [4], [5], [6], [7]:

- Measurement of spectral characteristics of satellite images
- Psychovisual examination of image compressing methods
- Qualification of potato seed and chips
- Temporal examination of damage of plant parts
- Classification
- Virtual Reality based 3D terrain simulation

4. Conclusion

When examining digital images where scales can be of great importance (image compressing, psychovisual examinations, printing, chromatic examinations etc.) SFD is suggested to be taken among the so far usual (eg. sign/noise, intensity, size, resolution) types of parameters (eg. compression, general characterization of images). Useful information on structure as well as shades can be obtained applying the two parameters together.

Several basic image data (aerial and space photographs) consisting of more than three bands are being used in practice. There are hardly any accepted parameters to characterize them together. I think SFD can perfectly be used to characterize (multi-, hyper spectral) images that consist of more than three bands.

On the basis of present and previous measurements it can be stated that SFD and FD are significant parameters in the classification of digital images as well.

SFD can be an important and digitally easily measurable parameter of natural processes and spatial structures besides the structural parameters used so far. These measurements are being carried out at present already - using the above method applicable in practice - and are to be accessed by anyone [16].

The applied method has proven that with certain generalization of the Box method fractal dimension based measurements – choosing appropriate measures- give practically applicable results in case of optional number of dimension.

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